ABSTRACT

There is a need to reliably predict the performance (efficiency and total pressure rise) of axial fans from model tests not only at the design point but also at part- and overload. The commonly used scale-up formulae give satisfactorily results only near the design point where inertia losses are small in comparison with frictional losses. At part- and overload the inertia losses are dominant and the scale-up formulae so far used fail. This is shown by applying several common scale-up methods in comparison to measured values of two axial model fans with a diameter of 1000 mm respectively 250 mm at different rotational speeds, hence Reynolds-numbers. In addition efficiency split-up for axial fans is shown and compared to former measurements on pumps.

NOMENCLATURE

\( A \) Area
\( D \) Diameter
\( H \) hydraulic head
\( M \) torque
\( M_a \) Mach number
\( P_h \) hydraulic power
\( P_m \) mechanical power
\( R_z \) roughness according to ASME Y14.36M - 1996
\( R_e \) Reynolds number: \( \pi D^2 n / \nu \)
\( \dot{V} \) volume flow rate
\( a \) speed of sound
\( n \) rotational shaft speed
\( p \) static pressure
\( p_t \) total pressure
\( p_d \) dynamic pressure
\( y \) specific work

Greek symbols
\( \eta \) efficiency: \( P_h / P_m \)
\( \kappa \) scale factor: \( D / D' \)
\( \lambda \) power coefficient: \( \frac{8 P_m}{\pi D^3 n^3} \)
\( \nu \) kinematic viscosity
\( \psi \) pressure coefficient: \( 2 \frac{\Delta p_t}{\rho \pi (D n)^2} \)
\( \rho \) density
\( \varphi \) flow rate coefficient: \( \frac{\dot{V}}{\pi / 4 D^2 n} \)

INTRODUCTION

Scale up formulae must be accurate enough to allow a realistic economic forecast at all operating points of a full scale turbo-machine in an early stage of the project. According to dimensional analysis the efficiency of a scaled model and the full scale machine (fs) is the same, as long as all nondimensional products are equal. For turbo-machines a full similarity between model (diameter \( D \)) especially at large scales (\( D' \)) is theoretical possible but unrealistic. The performance of the smaller model fan (sm) considered in this work, has an outer fan diameter of \( D = 250 \text{ mm} \), the full scale machine has an outer fan diameter of \( D' = 2500 \text{ mm} \) (see section Test Rigs). To have full similarity, i.e. \( R_e = R_{e}' \), between model and full scale turbo-machine,
the geometrical scaling factor $\kappa = D/D' = 0.1$ would require a scale factor of 100 for the rotational shaft speed and total pressure rise. The scale factor for the volume flow rate would be 0.1 and that for the mechanical power would be 10. Hence, for the turbo-machine considered in this work the model with diameter of 250 mm would have to run at 100 000 rpm and 4 MW.

**REVIEW OF COMMON SCALE-UP FORMULAE**

From the point of dimensional analysis the physical motivated scale-up formulae can be traced back to the work of Pfleiderer and Ackeret in mid of the 20th century. An overview can be found in [1, 2]. Some of the most common scale-up formulae are:

Moody I [3]

$$\frac{1 - \eta'}{1 - \eta} = \left(\frac{D}{D'}\right)^{0.25}, \quad (1)$$

Staufer [4]

$$\frac{1 - \eta'}{1 - \eta} = \left(\frac{D}{D'}\right)^{0.25} \left(\frac{H}{H'}\right)^{0.25} = \left(\frac{D}{D'}\right)^{0.25} \left(\frac{y}{y'}\right)^{0.25}, \quad (2)$$

Pfleiderer [5]

$$\frac{1 - \eta'}{1 - \eta} = \left(\frac{Re}{Re'}\right)^{0.1}, \quad (3)$$

Ackeret [6]

$$\frac{1 - \eta'}{1 - \eta} = V \left[1 + \left(\frac{Re}{Re'}\right)^{0.2}\right] \text{ with } V = 0.5, \quad (4)$$

Moody’s and Staufer’s formulae were one of the first published formulae that described efficiency scale-up. They are based only on empirical studies. For both the losses are not correlated with the inertia or friction forces on the blade. The friction losses were first treated in a truly physical motivated manner and on the basis of dimensional analysis by Pfleiderer: He assumed that the friction losses in the turbo-machine are proportional to a negative power of the Reynolds-number as it is observed for example in the Moody diagram for turbulent flow at an intermediate range of the Reynolds-number. He did not consider the limiting case of a constant friction factor at very high Reynolds-numbers. Due to the well known fact that the viscous sub-layer of the Prandtl-turbulent-wall-layer vanishes between the surface roughness, the influence of viscosity and thus the Reynolds-number vanishes. Also he did not take into account form losses which are truly inertia losses and thus independent of the Reynolds-number. The formula recommended in the latest German standard for final inspections of fans [7] is based on the Ackeret formula which is up to now the most accepted scale-up formula for fans in Europe. Ackeret divided the losses in a part that depends on viscosity effects and thus on the Reynolds-number and one that is independent of viscous friction. It was assumed by Ackeret that at the best efficiency point, where no losses derive from incidence, half of the losses are inertia losses and half are based on viscous friction, hence can be scaled-up [6].

Besides there are two fundamental problems all formulae have to face. Firstly all have in common that the efficiency could increase up to one, as long as the scale factor is big enough. (Several studies investigated the exponent of the Reynolds-number (e.g. [2, 8, 9]). These studies are drawing the conclusion, that the exponent is in the range of 0.1 . . . 0.25. Furthermore the exponent cannot be treated as a constant, but varies according to the range of considered Reynolds-number. Stoffel [10] introduced a modification of the Pfleiderer formula, were the limiting case of infinite Reynolds-number is covered by an efficiency which is smaller as one, as it must be.) Secondly all presented formulae assume complete geometric similarity. In most cases all parts are manufactured with the same technique, so the resulting roughness is nearly constant, however, geometric similarity requires equal relative roughness. The same problem arises in respect of tip clearance. Regarding the earlier mentioned example, the tip clearance of the $f_0$ is in the range of 3 mm. Scaled down to the model fan the tip clearance must be 0.3 mm high. Realising this small gap in practice is too costly, so even here geometric similarity is infringed.

**TEST RIGS**

This paper presents first results of a work with the objective to develop a new scale-up formula applicable to the complete operating range of axial fans, permitting different Reynolds-numbers, relative roughness and relative tip clearance on full scale machine and scaled model. Therefore two scaled axial fans, which are geometric similar to a full scale fume outlet fan used in a road tunnel in Lermoos, Austria, were planned, manufactured and assembled. Both allow a variation of the above mentioned parameters. All fans models and full scale machine have a blade adjustment. The basic data of the three fans is shown in tab. 1.

**Methodology**

**Volume Flow Rate** Figure 1 shows the configuration for volume flow rate measurement. A settling chamber with a grid
Table 1. BASIC DATA OF THE TWO MODEL FANS AND THE FULL SCALE FANS

<table>
<thead>
<tr>
<th>Name</th>
<th>small model (sm)</th>
<th>large model (lm)</th>
<th>full scale machine (fs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>250 mm</td>
<td>1000 mm</td>
<td>2500 mm</td>
</tr>
<tr>
<td>Speed</td>
<td>2475...6900 rpm</td>
<td>1240...2475 rpm</td>
<td>495, 990 rpm</td>
</tr>
<tr>
<td>Re</td>
<td>$0.5 \times 10^6$...$1.5 \times 10^6$</td>
<td>$4.3 \times 10^5$...$8.6 \times 10^5$</td>
<td>$11\times 10^5$, $22 \times 10^5$</td>
</tr>
<tr>
<td>Ma</td>
<td>0.09...0.27</td>
<td>0.19...0.38</td>
<td>0.19, 0.38</td>
</tr>
<tr>
<td>Rz (abs/rel)</td>
<td>9 µm / $36 \times 10^{-6}$</td>
<td>12 µm / $12 \times 10^{-6}$</td>
<td>52 µm / $21 \times 10^{-6}$</td>
</tr>
<tr>
<td>Gap (abs/rel)</td>
<td>0.25 mm / 1%</td>
<td>1 mm / 1%</td>
<td>3.3 mm / 1.3%</td>
</tr>
<tr>
<td>Power</td>
<td>4 kW</td>
<td>64 kW</td>
<td>400 kW</td>
</tr>
<tr>
<td>Diameter</td>
<td>250 mm</td>
<td>1000 mm</td>
<td>2500 mm</td>
</tr>
</tbody>
</table>

Figure 1. MEASURING POSITIONS FOR DETERMINING THE VOLUME FLOW RATE

and a honey-comb flow straightener downstream of the stage decreases remaining swirl and turbulence, so that a uniform flow is ensured. The volume flow rate is measured by a fixed Prandtl-probe, which was calibrated by the drafted comb-probe. The latter consists of $30/16$ ($lm/sm$) Pitot-tubes with decreasing distance near to the wall (minimum distance 1.5/1.7 mm). For better resolution of the stream it was rotated in $45^\circ/90^\circ$-steps over the circumference. To calculate the velocity by the dynamic pressure, the static pressure is measured in $60^\circ/90^\circ$-steps over the circumference in the same plane.

Pressure Scanners made by Pressure Systems are used. They allow a simultaneous measuring of all relevant pressures. Due to a previous calibration, the error in measurement could be decreased to less than 1Pa. The static pressure is measured in front of the rotor and behind the outlet guide vanes at the end of the diffusor (fig. 2). Total pressure rise is calculated by adding the dynamic pressure difference - calculated with the measured volume flow rate and the area ratio:

$$
\Delta p_t = \Delta p + \Delta p_d = p_5 - p_2 + \left(\frac{\dot{V}}{A_5^2} - \frac{1}{A_2^2}\right)
$$

Torque A torque sensor made by Manner Sensortelemetrie is used for torque measurement. The torque flange is mounted flying, so no bearing losses are measured. Due to the small friction surface, the disc friction loss is, compared to the other losses, negligible. It can be assumed that the hydraulic moment is measured directly.

Rotating Speed The rotating speed is measured with a tooth wheel (36 impulses per rotation) and an optical sensor.

Stagger angle Due to the blade adjustment it is possible to change the stagger angle in all fans (model and full scale machine). Besides the design angle ($\beta_s$) $\Delta \beta_s = -18, -12, -6$ and $+6$ were measured. A definition of the stagger angle can be found in fig. 3.

Error in Measurement The efficiency is calculated with

$$
\eta = \frac{\dot{V} \Delta p_t}{M^2 \pi n}
$$

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The error in efficiency is dominated by the volume flow rate and the torque measurement. It lies within 0.5...2% absolute efficiency, according to volume flow rate and torque. In figures showing the efficiency an errorbar is always plotted.

**COMPARISON OF MEASUREMENTS AND PREDICTED SCALE-UP**

Figures 6 to 10 show the measured efficiency compared with the scale-up of the presented formulae. Figure 4 displays the general proceeding: All points measured are interpolated with a 5th degree polynomial, which is used to appoint \( \varphi/\varphi_{\text{opt}} \) where the minimum Reynolds-number has its efficiency maximum. The increase of efficiency is examined along \( \varphi = \varphi_{\text{opt}} \) (Re\(_{\text{min}}\)). In addition the increase of efficiency at variable \( \varphi/\varphi_{\text{opt}} \) is shown as a function of Reynolds-number. The scale-up is in every case based on measured values at a Reynolds-number of \( 0.5 \times 10^6 \), which is the lowest curve in fig. 4 and fig. 5.

Each scale-up figure contains additionally to the results of this work the efficiency of the characteristic diagram of the \( \text{fs} \). It was extrapolated by the distributor on the basis of an own down-scaled model with \( \kappa \approx 0.22 \) and Re \( \approx 3.3 \times 10^6 \), efficiency was scaled-up with Ackeret’s formula. Though it was assumed that Ackeret’s formula supplies an efficiency scale-up that is much too small. Due to a constant measured efficiency above \( \text{Re} \approx 7.6 \times 10^6 \) the predicted efficiency approaches the measured and will reach the proper level beyond \( \text{Re} > 21.6 \times 10^6 \) (and will than increase further). In opposite Staufer’s formula fits for the lower rotation speed respectively Reynolds-number of the \( \text{sm} \) and overpredicts the efficiency for higher Reynolds-numbers in general (to calculate the needed specific work, a constant pressure coefficient is approached).

The result of the predicted scale-up is heavily depending on the reference Reynolds-number - e.g. starting at the maximum...
Reynolds-number of the $sm$, would lead to a different result. This approves that the exponent in formulae 2 to 4 is addicted to the considered Reynolds-number range, what Suspends these formulae from an universal use.

Moody’s formula only regards changes in diameter, nevertheless scale-up for the Reynolds-number range of the $fs$ shows a good trend, again assuming that the efficiency reaches a maximum at $Re\approx 7.6\times 10^6$. Due to the fact that the formula does not regard changes in Reynolds-number, it must be presumed that the result is coincidental.

**Part- and Overload** Considering the maximum efficiency at different Reynolds-numbers shows, that the best efficiency point (BEP) moves to higher $\varphi$ (fig. 4). According to this, the scale-up effect is expected to be higher at overload. Figures 7 and 8 affirm this. Whereas the maximum difference of efficiency at $\varphi/\varphi_{opt} = 1$ of the $sm$ and the $lm$ is roughly 9%, the difference increases at $\varphi/\varphi_{opt} = 1.2$ to roughly 14%. Looking at $\varphi/\varphi_{opt} < 1$ the opposed trend is visible - the maximum difference of efficiency decreases to roughly 5% at $\varphi/\varphi_{opt} = 0.85$.

Corresponding to the mentioned effects, the quality of efficiency prediction of the formulae differs. Due to general underprediction in case of Ackeret’s and Pfleiderer’s formulae, prediction quality improves for $\varphi/\varphi_{opt} < 1$ and worsen even more for $\varphi/\varphi_{opt} > 1$. While Staufer’s formula fits good for the observed Reynolds-number range at $\varphi/\varphi_{opt} = 1.1$ (beyond the range the scale-up will be too high again), it underpredicts the efficiency for $\varphi/\varphi_{opt} = 1.2$ and strongly overpredicts the efficiency for partload.

The comparison of the several scale-up formulae shows, that none of them is able to predict reliably the rise of efficiency, especially if different load ranges are regarded.

**Efficiency Split-up** The split-up of the efficiency increase was part of several former investigations concerning water turbines and pumps, e.g. Rotzoll [8]. Figure 11 shows the decrease of power coefficient with increasing Reynolds-number for a radial machine. It is obvious that the drop of disc friction power is fundamental for the appearing effect. Due to negligible disc friction power at the investigated setup in this paper, there is almost no drop of the power coefficient detectable. The increase of efficiency is only induced by a raising pressure coefficient (see fig. 12).

**SUMMARY**

An experimental test series is presented, which aimed to proof the accuracy of existing efficiency scale-up formulae. The rise of efficiency at different Reynolds-numbers were compared with the predicted of common scale-up formulae. It was shown that the formulae used so far are not able to dependably predict the increase of efficiency for the design point and especially for
different load ranges. Furthermore a breakdown of efficiency increase on pressure and power coefficient was presented. A de-
velopment to former experimental investigations showed a good agreement. This paper is part of a work at the chair of Fluid Systems Technology at the Technische Universität Darmstadt. It
aims on a new scale-up formula which not only includes infrac-
tion of kinematic similarity (Reynolds-number) but also sacri-
ficed geometric similarity (relative roughness, relative tip clear-
ance) which usually occur in practice.

ACKNOWLEDGMENT
The authors would like to thank the Arbeitsgemeinschaft in-
dustrieller Forschungsvereinigungen "Otto von Guericke" e.V.
(AIF), the Bundesministerium für Wirtschaft und Technologie
(BMWi), and the Forschungsvereinigung für Luft- und Trock-

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