ON RELIABLE PERFORMANCE PREDICTION OF AXIAL TURBOMACHINES

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ABSTRACT

There is a need to reliably predict the performance (efficiency and total pressure rise) of axial turbomachines from model tests for different load ranges. The commonly used scale-up formulas are not able to reliably predict the performance, especially beyond the design point. Furthermore, these formulas do not regard changes in rela- tive roughness as they usually occur in practice. An improved scale-up formula is proposed which achieves not only the reliable scale-up of efficiency, but also the scale-up of the pressure coefficient. It is motivated from measurements on two geometric similar axial model fans with a diameter of 1000 mm respectively 250 mm at different rotational speeds, hence Reynolds numbers. By bonding grains of sand to the im- peller the influence of relative roughness was investigated. For applying the formula to different load ranges a factor $V$ is intro- duced that depends on the quotient of effective flow coefficient to optimal flow coefficient.

INTRODUCTION

Acceptance inspection on large axial turbomachines with impeller diameters of several meters are carried out on the manufacturers own laboratory because of good accessibility, an improved inflow and the possibility to use better measuring tech- nology. Downscaled models are used due to high manufacturing costs, huge required space and excessive installed capacity in case of using full scale machines (also upscaled models might be used in order to reduce costs by having only one standard-
are identical. The model machine (sm) considered in this work, shows an outer rotor diameter of \(d_{\text{sm}} = 250 \text{ mm}\), the full scale machine has an rotor diameter of \(d_{\text{fs}} = 2500 \text{ mm}\) (see section Test Rigs). To have similarity in the Reynolds number, the scaling factor \(\kappa = d_{\text{sm}}/d_{\text{fs}} = 0.1\) would require a scale factor of 100 for the rotational shaft speed and total pressure rise. The scale factor for the volume flow rate would be 0.1 and that for the mechanical power 10. Hence, the considered model with a diameter of 250 mm would have to run at a rotational speed of 100 000 rpm and would require an input power of 4 MW. This leads to a circumferential Mach number of \(M_t \sim 4\) and a volume specific power of 1/64 \(\text{GW/m}^3\) which is unachievable. To overcome that problem the similarity in the Reynolds number is given up and the total pressure rise is kept constant. Hence the Reynolds number scale is \(R_{\text{es}} = \kappa R_{\text{es}}\), the Mach number \(M_{\text{es}} = M_{\text{es}}\). For the experiments the rotational speed and the power input is reduced to 10 000 rpm respectively 4 kW in the mentioned example. Due to given up similarity an efficiency and pressure rise scale-up is necessary.

**INFLUENCE OF REYNOLDS NUMBER**

The influence of Reynolds number is a largely examined correlation for geometries such as pipes, flat plates, etc. Since for all turbulent flow near a wall the findings of Prandtl and von Kármán are valid also for the relative flow along the impeller blades, it is reasonable (see [2]) to use the resistance law of either the flat plate (Nikuradse, Schlichting) or the resistance law for the turbulent pipe flow which is shown in Fig. 1, stressing the two limiting cases (i) the hydraulically smooth and (ii) the hydraulically rough limit. For the first case the typical roughness length \(k\) or \(k_s\) is smaller than the viscous length

\[
\frac{k}{\delta_\nu} = \frac{u_* k}{\nu} \sim Re k d^{0.5 < 5}. \quad (8)
\]

Thereby the exact (see [3]) or approximated power law behaviour (the empirical Blasius law \(\alpha = 1/4\))

\[
\lambda \sim Re^{-\alpha} \quad (9)
\]

holds. For the second case the typical roughness length \(k\) is much greater than the viscous length. For fully developed turbulent pipe flows where the endwall boundary layers have merged the exact result is
\[ \lambda = \left( 2 \log \frac{d}{2k} + 1.74 \right)^{-2}. \tag{10} \]

One may argue that the fully developed turbulent pipe flow is physically different from the flow along an impeller blade. But in the region of viscous length \( \delta_v \) they are similar. Since only ratios of friction factors are important for scaling the findings of Spurk [2], i.e. similar results for upscaling on the basis of a turbulent flow along a flat plate or fully developed pipe flow are identical.

Scaling viscous friction losses, i.e. Reynolds number dependent losses, are usually named scalable losses. The independent losses, i.e. inertia losses, are named non scalable losses. In the context of axial turbomachines the ideal pressure rise is given by

\[ \Delta p_{\text{h}} = \Delta p_{\text{ideal}} - \Delta p_{\text{loss}}. \tag{11} \]

or likewise in the dimensionless form

\[ \psi = \psi_{\text{ideal}} - \psi_{\text{loss}}. \tag{12} \]

Since the performance of a measured machine is known in form of the functions \( \psi(\varphi, Re, Ma, k/d, t/d) \) and \( \eta(\varphi, Re, Ma, k/d, t/d) \) the characteristic of the ideal machine follows from:

\[ \psi_{\text{ideal}}(\varphi) = \psi_0 + \psi_0' \varphi = \frac{\psi(\varphi, Re, Ma, k/d, t/d)}{\eta(\varphi, Re, Ma, k/d, t/d)} \tag{13} \]

This is valid for axial machines and can be confirmed by own measurements shown in Fig. 10,15. The constants \( \psi_0 \) and \( \psi_0' \) are determined only by dimensionless geometrical parameters such as stagger angle \( \beta_s \) (see Fig. 6) and the number

\[ N := 1 - \left( \frac{r_i}{r_o} \right)^2, \tag{14} \]

where \( r_i \) denotes the hub radius and \( r_o \) the tip radius. Following Eqn. (13) and confirmed by the experimental findings (see Fig. 8, 9, 10) the total losses follow from \( \eta, \psi \) by

\[ \psi_{\text{loss}} = \psi - \frac{1}{\eta}. \tag{15} \]

The measurements show a shift of the best efficiency point (BEP) with increasing Reynolds number to higher values of the flow rate (see Fig. 8). This behaviour is explained by a split of total losses into friction losses \( \psi_t \) and losses caused by incidence and tip clearance, which are independent of the Reynolds number:

\[ \psi_{\text{loss}} \approx \psi_t(Re, \varphi, k/d) + \psi_i(\varphi) + \psi_t(\varphi, t/d). \tag{16} \]

Implicit with Eqn. (16) it is assumed that all types of losses are independent of each other which is not entirely accurate. In fact the split-up of losses is similar to the Froude hypothesis used to scale up towing experiments needed ship engineering.

**REVIEW OF COMMON SCALE-UP FORMULAS**

The common physically based scale-up formulas are based on the ideas of Pfleiderer and Ackeret (an overview of scale-up formulas can be found in [4, 5]). They are special forms of the here given general form:

\[ 1 - \frac{\eta_{fs}}{\eta_{fm}} = 1 - V(\varphi) \left\{ 1 - \frac{\lambda[Re_{fs}, (k/d)_{fs}]}{\lambda[Re_{m}, (k/d)_{m}]} \right\}, \tag{17} \]
where \( V(\varphi) \) denotes a load dependent function which usually is proposed to have a constant value of 0.5 [6], [7]) or covering the range from 0.3 to 1 [8], [9]).

Following this discussion the well known Pfleiderer formula [10] follows from the general Eqn. (17) by setting \( V = 1 \) and assuming hydraulically smooth turbulent flow (contrarily to the Blasius law he used an exponent of 0.1):

\[
\frac{1 - \eta_{fs}}{1 - \eta_{m}} = \left( \frac{Re_{fs}}{Re_{m}} \right)^{0.1}.
\]

Ackeret was the first researcher who made a distinction between friction losses (Reynolds dependent, scalable) and inertia losses (Reynolds independent, not scalable) [6]. He assumed \( V = 1/2 \) and hydraulically smooth surface. This leads to a formula which is applied today only in the BEP

\[
\frac{1 - \eta_{fs}}{1 - \eta_{m}} = \frac{1}{2} \left( 1 + \left( \frac{Re_{fs}}{Re_{m}} \right)^{-0.2} \right).
\]

Alternatively to Eqn. (17) a physical motivated method can be employed or a taylor expansion of \( \eta \) and \( \psi \) using either model data or physical results such as Fig. 1 can be employed. This was proposed by Spurk [3]. Over the here considered wide range of relative roughness this approach was not successful. The attraction of the approach by Spurk is that no model has to be used or that an empirical function \( V(\varphi) \) is needed. Both approaches are limited to a specific type of turbomachines such as axial machines in this case.

The authors have developed two models [11]) where the first one tries to determine \( V(\varphi) \) completely analytical. Figure 2 shows a significant improvement in the scaling of the efficiency by using the analytical way to determine the loss distribution factor in comparison to a fixed value of 0.5 as Ackeret did. The effect of shifting BEP to higher flow rate coefficients would not be covered using the introduced method. The second method in the mentioned work is based on a friction model which leads to a result shown in Fig. 3. The calculated values are still smaller than the measured ones but an increase of accuracy compared to Ackeret is obtained.

Since none of the mentioned methods are able to reliably predict the efficiency and especially the shift of BEP this paper wants to employ the more general formula (17) to give a scale-up method which takes into account the surface roughness and the operating point.

\[
\frac{1 - \eta_{fs}}{1 - \eta_{m}} = \frac{1}{2} \left( 1 + \left( \frac{Re_{fs}}{Re_{m}} \right)^{-0.2} \right).
\]

TEST RIGS

The improved formula is based on measurement on two scaled axial fans, which are geometric similar to a full scale smoke extraction fan used in a road tunnel in Lermoos, Austria. Both models allow a variation of rotational speed, hence Reynolds number, and stagger angle \( \beta_s \). Due to infringed similarity, different stagger angles are analog to measurements on different machines. The basic data of the three fans are given in Tab. 1. The rotor blades of the models are made of high-strength aluminum. Prior tests with carbon fiber laminate lead to a change in stagger angle of the airfoils due to centrifugal forces - the effect of scale-up shown here is only based on Reynolds number influence.
The table below shows the basic data of full scale and model fans:

<table>
<thead>
<tr>
<th>Name</th>
<th>small model (sm)</th>
<th>large model (lm)</th>
<th>full scale machine (fs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>250 mm</td>
<td>1000 mm</td>
<td>2500 mm</td>
</tr>
<tr>
<td>Speed</td>
<td>2800 . . . 6900 rpm</td>
<td>1240 . . . 2475 rpm</td>
<td>495, 990 rpm</td>
</tr>
<tr>
<td>Re</td>
<td>0.1 . . . 1.5 E06</td>
<td>4.3 . . . 8.6 E06</td>
<td>11, 22 E06</td>
</tr>
<tr>
<td>Ma</td>
<td>0.11 . . . 0.27</td>
<td>0.19 . . . 0.38</td>
<td>0.19, 0.38</td>
</tr>
<tr>
<td>$R_e$ (abs/rel)</td>
<td>9 $\mu$m / 36 E-06</td>
<td>12 $\mu$m / 12 E-06</td>
<td>52 $\mu$m / 21 E-06</td>
</tr>
<tr>
<td>Gap (abs/rel)</td>
<td>0.25 mm / 1%</td>
<td>1 mm / 1%</td>
<td>3.3 mm / 1.3%</td>
</tr>
<tr>
<td>Power</td>
<td>4 kW</td>
<td>64 kW</td>
<td>400 kW</td>
</tr>
</tbody>
</table>

Table 1. BASIC DATA OF FULL SCALE AND MODEL FANS

Methodology

**Volume Flow Rate**  Figure 4 shows the configuration for volume flow rate measurement. A settling chamber with a grid and a honey-comb flow straightener downstream of the stage decreases remaining swirl and turbulence, so that a uniform flow is ensured. Due to less available space for a flow-calming section required for using a venturi tube or orifice plate the volume flow rate was measured by a fixed Prandtl-probe, which was calibrated by the drafted comb-probe. The latter consists of 30/16 (lm/sm) Pitot-tubes with decreasing distance near to the wall (minimum distance 1.5/1.7 mm). For better resolution of the stream it was rotated in 45°/90°-steps over the circumference. To calculate the velocity by the dynamic pressure, the static pressure is measured in 60°/90°-steps over the circumference in the same plane. The resulting averaged flow profile was expanded by two additional points near the wall by using the 1/7 power law [3]. Thereby the parameters were adjusted to fit the profile of the nearest measuring points. The arising profile covers 99.7% of the effective cross sectional area.

**Pressure**  Scanners made by Pressure Systems were used. They allow a simultaneous measuring of all relevant pressures. Due to a previous calibration, the error in measurement could be strongly decreased (see Tab. 3). The static pressure is measured in front of the rotor and behind the outlet guide vanes at the end of the diffuser (Fig. 5). Total pressure rise is calculated by adding the dynamic pressure difference - calculated with the measured volume flow rate and the area ratio:

$$\Delta p_t = p_{s,3} - p_{s,1} + \frac{\rho}{2} \frac{Q^2}{A_2^2} \left[1 - \left(\frac{A_4}{A_1}\right)^2\right]$$

Torque  A torque sensor made by Manner Sensortelemetrie was used for torque measurement. The torque flange is mounted flying (with integrated telemetry), so no bearing friction torque is measured. Due to the small friction surface, the disc friction torque is, compared to the other torques, negligible. It can be assumed that the hydraulic torque is measured directly.

Rotating Speed  The rotating speed is measured with a tooth wheel (36 impulses per rotation) and an optical sensor.

Stagger angle  Due to the blade adjustment it is possible to change the stagger angle in all fans, i.e. model and full scale machine. Besides the design angle $\beta_s$, $\Delta \beta_s$ was changed by -6° and -12° (see Fig. 6). A definition of the stagger angle can be found in Fig. 6.

Roughness  The impeller blades are micro-blasted with a $R_z$-roughness of 9/12 $\mu$m (sm/lm). Table 2 shows the grain diameters to modify the surface roughness. It also lists the tolerance in diameter and the measured $R_z$-values (each surface roughness was measured on different places on the blade and parallel/perpendicular to the flow direction - no dependency on flow direction could be assessed). There is a wide range of conversation factors for the relation of technical and sand roughness described in literature (an overview can be found in [5]). Based on own roughness measurements an approximate relation is done by using the scaling $R_z = k_s \cdot 1.5$.  

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The error caused by calibration implies geometrical errors (paraxial tolerance, flashes on pitot tubes as well as radial offset of the pitot tubes - influences are estimated) and otherwise not mentioned errors (influence of wall treatment, approximation of measured values by 5th degree polynomial and variation of ambient conditions while calibration at different comb-probe angels - influences are estimated). By using the calibration protocol the remaining error of the pressure scanner (affects all pressure measurements except ambient pressure $p_{0}$) and utilizing a calibration curve for measurement analysis the error could be highly decreased. The specified remaining error is primarily based on temperature sensitivity and zero drift. The latter was controlled by measuring the difference to the ambient pressure before and after each characteristic curve - the maximum difference of 0.2 Pa is also covered by the indicated error.

The error in torque measurement consists of variance in linearity and the error based on different temperatures at calibration and measurement. The latter is only affected by the difference in temperature, the variance in linearity is strongly depending on the effectively indicated value. According to this a curve was used for measurement analysis (exemplary Fig. 7), that allows a calculation of the actual error. The presented calibration was performed by applying static forces what leads to a hysteresis effect - due to dynamic load in the measurement setup the maximum error of the hysteresis curve was always used.

### Table 3. BIAS IN MEASURED VALUES

<table>
<thead>
<tr>
<th>Measured Variable</th>
<th>Bias $sm$</th>
<th>Bias $lm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient pressure $p_0$ in Pa</td>
<td>±15</td>
<td>±20</td>
</tr>
<tr>
<td>Ambient temperature $t$ in °C</td>
<td>±1</td>
<td>±1.5</td>
</tr>
<tr>
<td>Inner radius $r_i$ in mm</td>
<td>±0.15</td>
<td>±0.2</td>
</tr>
<tr>
<td>Outer radius $r_{o,1}/r_{o,4}$ in mm</td>
<td>±0.05/0.15</td>
<td>±0.2/0.2</td>
</tr>
<tr>
<td>Pressure scanner in Pa</td>
<td>±0.5</td>
<td>±0.5</td>
</tr>
<tr>
<td>$n$ in %</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$M$ in Nm</td>
<td>variable</td>
<td>variable</td>
</tr>
<tr>
<td>Error due to calibration in %</td>
<td>±0.4</td>
<td>±0.4</td>
</tr>
</tbody>
</table>

### Table 4. BIAS IN CALCULATED PARAMETERS

<table>
<thead>
<tr>
<th>Calculated Parameter</th>
<th>$\Delta \beta_i$ = 0°</th>
<th>$\Delta \beta_i$ = -6°</th>
<th>$\Delta \beta_i$ = -12°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$ in %</td>
<td>&lt; 1%</td>
<td>&lt; 1.2%</td>
<td>&lt; 1.5%</td>
</tr>
<tr>
<td>$\Delta p_{i}$ in %</td>
<td>&lt; 1%</td>
<td>&lt; 1.1%</td>
<td>&lt; 1.6%</td>
</tr>
<tr>
<td>$\eta$ in %</td>
<td>&lt; 1.9%</td>
<td>&lt; 2.5%</td>
<td>&lt; 3.3%</td>
</tr>
</tbody>
</table>

### Evaluation of measurements

Figure 8 displays the general proceeding in measurement analysis: All points measured are interpolated with a 5th degree polynomial, which is used to appoint $\varphi_{opt}$ where the minimum Reynolds number has its efficiency maximum. The increase of efficiency is examined along $\varphi = \varphi_{opt}(Re_{min})$. The scale-up is...
in every case based on measured values at a Reynolds number of 0.5 × 10^6, which is the lowest curve in Fig. 8 and Fig. 9.

**IMPROVED FORMULA**

The improved formula uses the general scale-up formula in Eqn. (17) with the following adaptations:

**Exponent \( \alpha \)**

The exponent in the Ackeret formula is based on the empirical Blasius law for flat plates \( \alpha = 1/5 \) [3]. In axial fans friction losses are also generated on end wall and hub so an adaption is required. The losses occurring in the fan were not measured directly but they can be calculated by using Eqn. (15). The ideal pressure coefficient is independent from changes in Reynolds number, whereas the pressure loss coefficient is changing with varied Reynolds number as shown in Fig. 10.

The trend of pressure loss coefficient for variable \( \psi \) for different Reynolds numbers at \( \Delta \beta_s = 0 \) is shown in Fig. 11. Two different exponents are displayed - the one that is used by Ackeret \( \alpha = -0.2 \) and the best correlation for the three measured stagger angles which is \( \alpha = -0.25 \).

**Critical Reynolds number \( Re_c \)**

As already mentioned, there are three bands of Reynolds number dependency. Modeling the zone between hydraulically smooth and hydraulically rough is possible but complex. Since industrial application of a scale-up formula should be simple and
easy to understand it necessitates simplification. As shown in Fig. 12 the real progression will be approximated to only two bands - one where an increase of Reynolds number influences the loss coefficient and - after overstepping a critical Reynolds number - another where the loss coefficient is independent. In the context of scaling only relative changes have to be considered, so the ratio of friction factors \( \lambda \) in Eqn. (17) can be replaced by the quotient of the regarded Reynolds numbers. The influence of roughness is regarded by introducing a critical Reynolds number \( R_{c} \), so the ratio of friction factors \( \lambda \) can be written as

\[
\frac{\lambda_{\text{meas}}}{\lambda_{\text{Ack}}}(Re, k/d) \approx \frac{\lambda_{\text{meas}}}{\lambda_{\text{meas}}}(R_{c}, k/d) \approx \lambda_{\text{meas}}(R_{c}, k/d) \text{approx.} \lambda_{\text{meas}}(R_{c}, k/d)
\]

\[\text{approx. } \lambda(Re, k/d)\]

The factor 1100 results from the measurements and is a best fit for all measured relative roughnesses and stagger angels. Table 5 and Fig. 13 compare the measured efficiencies at different Reynolds numbers and with varied relative roughness to the scale-up calculated by using the Ackeret and the improved formula. Besides the calculated efficiency according to Ackeret and the improved formula the gain in accuracy to Ackeret is shown.

<table>
<thead>
<tr>
<th>( Re )</th>
<th>( l/R_{z} )</th>
<th>( R_{c} )</th>
<th>( \eta_{\text{meas}} )</th>
<th>( \eta_{\text{Ack}} )</th>
<th>( \eta_{\text{imp}} )</th>
<th>gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.61E06*</td>
<td>2778</td>
<td>3.06E06</td>
<td>0.802</td>
<td>0.802</td>
<td>0.802</td>
<td>0%</td>
</tr>
<tr>
<td>0.61E06</td>
<td>1316</td>
<td>1.45E06</td>
<td>0.780</td>
<td>0.802</td>
<td>0.802</td>
<td>2.2%</td>
</tr>
<tr>
<td>0.61E06</td>
<td>329</td>
<td>0.36E06</td>
<td>0.767</td>
<td>0.802</td>
<td>0.780</td>
<td>2.2%</td>
</tr>
<tr>
<td>1.08E06</td>
<td>2778</td>
<td>3.06E06</td>
<td>0.825</td>
<td>0.813</td>
<td>0.823</td>
<td>1%</td>
</tr>
<tr>
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<td>1316</td>
<td>1.45E06</td>
<td>0.810</td>
<td>0.813</td>
<td>0.823</td>
<td>-1%</td>
</tr>
<tr>
<td>1.08E06</td>
<td>329</td>
<td>0.36E06</td>
<td>0.767</td>
<td>0.813</td>
<td>0.780</td>
<td>3.3%</td>
</tr>
<tr>
<td>1.51E06</td>
<td>2778</td>
<td>3.06E06</td>
<td>0.835</td>
<td>0.819</td>
<td>0.834</td>
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</tr>
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<td>1316</td>
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<td>0.815</td>
<td>0.819</td>
<td>0.833</td>
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<td>329</td>
<td>0.36E06</td>
<td>0.767</td>
<td>0.819</td>
<td>0.780</td>
<td>3.8%</td>
</tr>
<tr>
<td>4.32E06</td>
<td>8333</td>
<td>9.17E06</td>
<td>0.870</td>
<td>0.834</td>
<td>0.864</td>
<td>2.9%</td>
</tr>
<tr>
<td>4.32E06</td>
<td>2083</td>
<td>2.29E06</td>
<td>0.856</td>
<td>0.834</td>
<td>0.847</td>
<td>1.3%</td>
</tr>
<tr>
<td>4.32E06</td>
<td>1316</td>
<td>1.45E06</td>
<td>0.835</td>
<td>0.834</td>
<td>0.833</td>
<td>-0.1%</td>
</tr>
<tr>
<td>8.64E06</td>
<td>8333</td>
<td>9.17E06</td>
<td>0.886</td>
<td>0.843</td>
<td>0.879</td>
<td>3.6%</td>
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<tr>
<td>8.64E06</td>
<td>2083</td>
<td>2.29E06</td>
<td>0.856</td>
<td>0.843</td>
<td>0.847</td>
<td>0.4%</td>
</tr>
<tr>
<td>8.64E06</td>
<td>1316</td>
<td>1.45E06</td>
<td>0.835</td>
<td>0.843</td>
<td>0.833</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

Table 5. COMPARISON OF MEASURED AND CALCULATED EFFICIENCIES - REFERENCE IS MARKED WITH *

There are three cases in which the Ackeret formula gives better results than the improved formula. This circumstance occurs always when the Ackeret formula is very close to the measured efficiency what - as mentioned - considers to the regarded Reynolds number and relative roughness of model and full scale machine. But overall the maximum difference in predicted efficiency of the improved formula compared to the measured values is less than 2%, whereas the maximum of the efficiency calculated using the Ackeret formula to the measured is 5.2% \((Re = 1.51E06, 1/R_{z} = 329)\). The table shows, that for high values of \( l/R_{z} \) as they usually occur in full scale machines due to large diameters respectively chord lengths Ackeret strongly underpredicts the increase in efficiency. This is an important information for manufacturers of axial turbomachines, who probably sold their machines at less than fair value so far.
Load depending $V$

A fundamental problem of the Ackeret formula is the restriction of application to the BEP. Beyond this point every manufacturer has created its own approach, which makes it almost impossible for operators to compare machines from different companies with each other. Axial turbomachines with large diameters are often driven far away from the design point, e.g. fans for power plants which only runs in BEP on hot days when the plant has its maximum load or the considered $f$s of this work, which operates in BEP only in case of fire. Conditions for work in the BEP are seldom reached, so a scale-up for different load ranges than the BEP is strictly required. Based on formula 24 and the approach with $\alpha = -0.25$ and Reynolds numbers in the hydraulic smooth area a factor $V$ can be defined according to:

$$V (\varphi) = \frac{\frac{1}{1-\eta_m(\varphi)} - 1}{\frac{1}{R_{cm}} - 0.25} - 1$$  \hspace{1cm} (23)

Figure 14 shows all $V \left( \frac{\varphi}{\varphi_{opt}} \right)$ for the three stagger angles (the reference curve is in each case the respective lowest Reynolds number of each stagger angle). $V (\varphi/\varphi_{opt} = 1) = V_{opt}$ represents the fraction of losses which is based on friction, hence needed to be scale-up. Whereas a $V_{opt} = 0.8$ is used in the improved formula, the Ackeret formula in which $V_{opt}$ is also the fraction of friction losses presumes a value of only 0.5. Appraising the losses occurring in axial turbomachines, the value of the improved formula is reasonable, as $\approx 10\%$ fall upon tip clearance losses, the other $\approx 10\%$ can be related to remaining incidence losses (the stator blades are unshrouded, so it must be presumed that an incident generated mass always occurs) and diffusor losses.

The presumed value of 0.5 by Ackeret is underrated. Figure 14 shows values for $V (\varphi/\varphi_{opt} = 1) = V_{opt}$ above 1 which is physically impossible. This is owed to the factor $\alpha$, which is a compromise to the best fit for all measured stagger angles. In addition the appointed exponent $\alpha$ is most likely already in the zone of $\frac{h}{d} > 5$ (see Eqn. 8). For Reynolds numbers below $Re < 1.3E06$ the scale-up effect is greater than depicted by $\alpha = 0.25$ which is expressed by higher values of $L$. Since industrial models usually show a Reynolds number above $Re = 1.5E06$ ($n = 3000$ rpm, $d > 0.4$ m) this area is of subordinate interest.

$$V (\varphi/\varphi_{opt} \not= 1)$$ does not quantify the scalable losses in opposite to $V_{opt}$. Instead it specifies the influence due to the moving optimum which can be seen regarding the measured efficiencies in Fig. 8.

For industrial application it is necessary to find a common formula with a characteristic of $V (\varphi/\varphi_{opt})$ that is close to reality but never overestimates the efficiency in order to avoid contract penalties. Thus the characteristic in eq. 26 is used for the improved formula. For $\varphi/\varphi_{opt} < 1$ the scale-up effect decreases continuously for smaller values of $\varphi/\varphi_{opt}$, whereas it increases for $\varphi/\varphi_{opt} > 1$ and reaches a maximum for $\varphi/\varphi_{opt} = 2$ as shown in Fig. 14.

The former introduced adaptations for the general formula lead to the following improved formula for scale-up of efficiency and pressure coefficient:

$$\begin{align*}
\eta (\varphi) &= 1 - [1 - V (\varphi)] [1 - \eta_m (\varphi)] - \\
V (\varphi) [1 - \eta_m (\varphi)] \left( \frac{Re_s}{Re_m} \right)^{-0.25}
\end{align*}$$  \hspace{1cm} (24)

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\[ \text{Re}^*, \text{Re}^*_m = \begin{cases} \text{Re}^*, \text{Re}^*_m < \text{Re}_c, & \text{Re}^*, \text{Re}^*_m < \text{Re}_c, \\ \text{Re}^*, \text{Re}^*_m > \text{Re}_c, & \text{Re}^*, \text{Re}^*_m > \text{Re}_c \end{cases} \]

with \( \text{Re}_c = 137.5 \times \frac{l}{R_z} \)  

(25)

\[ V(\phi) = \begin{cases} \frac{\phi}{\phi_{\text{opt}}} < 1, & V_{\text{opt}} \left( \frac{\phi}{\phi_{\text{opt}}} \right)^4 \\ \frac{\phi}{\phi_{\text{opt}}} = 1, & V_{\text{opt}} \\ \frac{\phi}{\phi_{\text{opt}}} > 1, & V_{\text{opt}} + 0.4 \left[ 1 - \left( 2 - \frac{\phi}{\phi_{\text{opt}}} \right)^6 \right] \end{cases} \]

(26)

**Scale-up of the pressure coefficient**

The split-up of the efficiency increase of axial turbomachines was already investigated by Hess and Pelz [13]. Figure 15 shows the results for all three measured stagger angles \( \Delta \beta_s = 0, -6 \) and \(-12\). The correlation of increased pressure rise and efficiency can be clearly seen - whereas the power coefficient varies in the range of the accuracy of measurements the increase of efficiency is equivalent to the rise of pressure coefficient. This implicates, that the presented scale-up formula can also be used for calculating the increase of pressure coefficient in the form:

\[ \frac{\psi_{fs}}{\psi_{m}} = \frac{\eta_{fs}}{\eta_{m}} \rightarrow \psi_{fs} = \frac{\eta_{fs}}{\eta_{m}} \psi_{m}. \]

(27)

**SUMMARY**

An improved scale-up formula based on measurements on two geometric similar fans with modifiable relative impeller roughness is presented. Three different stagger angles were measured which is - due to infringed geometric similarity - analogue to measurements on three different machines.

It allows to calculate the rise of efficiency occurring due to increased Reynolds number. The improved formula regards the relative roughness of the impeller whereby an increase of precision is reached compared to the status quo formula derived from Kaplan-turbines by Ackeret in the 1940s. It is shown that high values of \( l/R_z \) - as they usually occur in full scale machines due to large diameters respectively chord lengths - the status quo strongly underpredicts the increase in efficiency. All the signs are that manufacturers of axial fans probably sold their machines at less than fair value so far.

![Figure 15. SPLIT-UP OF EFFICIENCY INCREASE ON POWER AND PRESSURE COEFFICIENT FOR DIFFERENT STAGGER ANGLES](image.png)

In addition a scale-up can now be performed not only in the best efficiency point, but also at different load ranges. Furthermore the important and so far not treated scale-up of the pressure coefficient is possible by using the same formula as for the efficiency.
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